



# Power control of optical CDMA star networks

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Received 21 February 2005; received in revised form 12 August 2005; accepted 28 September 2005

## Abstract

In this paper, we apply the power control concept to optical CDMA star networks. Two approaches are considered, namely, centralized and distributed power control. Both approaches are used to optimize the optical transmit power and to maximize network capacity in terms of the number of users satisfying a target signal to interference (SIR) ratio. Centralized algorithms result in the optimum power vector while distributed algorithms are more suitable for practical system implementation and eliminate the need for a centralized control node. Both analytical and simulation results show significant improvement in the performance of the power controlled optical CDMA system. For instance, in a network of 31 nodes, a doubling of the capacity as compared to the non power control case is obtained. Furthermore, we show in the interference-limited case that the network performance is upper bounded by the number of nodes and the correlation properties of the employed code rather than network attenuation and optical fiber lengths. The concept of network partitioning is then introduced to simplify optimum power calculations. Using network partitioning, we find in the interference-limited case that the optical fibers after the star coupler are irrelevant to the optimum power evaluation.

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PACS: 42.79.S

Keywords: Power control; Optical CDMA; Prime codes

## 1. Introduction

Since the mid-1980s when the single-mode fiber-optic media was seen to become a main solution of future telecommunications networks for transporting high-volume, high-quality, multimedia information, the need for all-optical multi-access networking became important. An all-optical multi-access network is a collection of multiple nodes where the interconnection among various nodes is via single- or multi-mode fiber optics and for which they perform all their essential signal processing functions (e.g., switching, add-drop, multiplexing/demultiplexing and amplification) in the optical domain. Optical CDMA (OCDMA) networking

is one possible technique that allows multiple users in local area networks to access the same fiber channel asynchronously with negligible amount of delay and scheduling. In contrast to optical time division multiple access (OTDMA), and wavelength division multiple access (WDMA), where maximum transmission capacity is determined by the total number of allocated time slots and wavelengths (i.e., hard-limited capacity), OCDMA allows more flexible network access because the bit error rate (BER) depends on the number of active users (i.e., soft-limited capacity) [1]. A variety of approaches for implementing OCDMA have been suggested in the literature [2–4]. All of these schemes share a common strategy of distinguishing data channels not by wavelength or time slot only, but also by distinctive spectral or temporal code (or signature) impressed onto the bits of each channel. Suitably designed receivers isolate channels by code-specific detection.

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In OCDMA, other users accessing the network at the same time as the desired user give rise to multi-access interference (MAI). This MAI can be high enough to make the LAN useless if the users' codes are highly correlated and the code weights are small as in conventional temporal intensity based optical coding techniques. Optical orthogonal codes (OOCs) were proposed for OCDMA with minimum auto- and cross-correlation values [5–7]. However, the OOC is a highly sparse code and the number of supported users can be very low. Another important type of code proposed for OCDMA is the prime code [8–11]. The prime code has a higher correlation value as compared to the OOC but it can support more users. Furthermore, the ease of construction and generation of prime codes makes them a good candidate for OCDMA networks [12,13]. Other factors affecting the performance of an OCDMA network are shot noise and thermal noise in the transceiver. These impairments can be neglected in optical local area networks (LANs) that apply a higher level of transmission power and optical preamplifiers in front of the receivers. Therefore, performance of LANs is limited basically by MAI and amplified spontaneous emission (ASE) generated by the optical amplifiers. However, MAI in OCDMA networks introduces the near-far problem, thus optical power control is needed to alleviate the problem and enhance the performance and throughput of the network.

Optical power control for OCDMA networks was extensively discussed in [14–16]. In [14], optical power control and Time Hopping for multimedia applications using single wavelength was proposed. The approach accommodates to various data rates using only one sequence by changing the time-hopping rate. However, to realize such system an optical selector device that consists of a number of optical hard-limiters is needed [14]. Unfortunately, the optical hard-limiters are not yet mature for field deployment. Power control has also been considered for optical fast frequency hopping CDMA to provide quality of ser-

vice compatibility by applying variable attenuators [15,16]. It has been shown that a great improvement in the system capacity is given by the power control but the influence of network impairments such as the nonuniform attenuation due to the difference in fiber lengths were not considered [22]. Disadvantages of this scheme include the need for multi-wavelength transmitters and susceptibility to wavelength-dependent impairments.

In this paper, we apply the power control to a pre-amplified temporal Prime coded OCDMA system taking into account the effect of fiber lengths, and the ASE noise. The paper is organized as follows: In Section 2, the system configuration based on prime codes is presented. In Section 3, optimum optical power levels are computed using a centralized power control algorithm. Then a partitioning of the network to access and broadcast parts is considered. The use of distributed algorithms is then discussed. The paper is concluded by summarizing results and remarks.

## 2. System description

A typical fiber optic CDMA system is shown in Fig. 1, where the nodes are connected through a passive  $K \times K$  star coupler, where  $K$  denotes the number of network access points. At the logical level this configuration is a broadcast-and-select network. Other network topologies can be used for OCDMA such as bus and ring topologies. There is no global optimum topology for fiber optic LAN interconnection [17]. Each of them has its own advantages and disadvantages, whose significance depends on the specific application under consideration. Let us consider Fig. 1, where all users are connected to the central star coupler using optical fiber links. Each user has two fiber cables, one for transmission and the other for reception. Or one could use only one fiber cable that has up and down links separated by different laser wavelengths. We neglect fiber dispersion (dispersion shifted fibers) and nonlinearity effects. If the node distances from the star coupler are quite

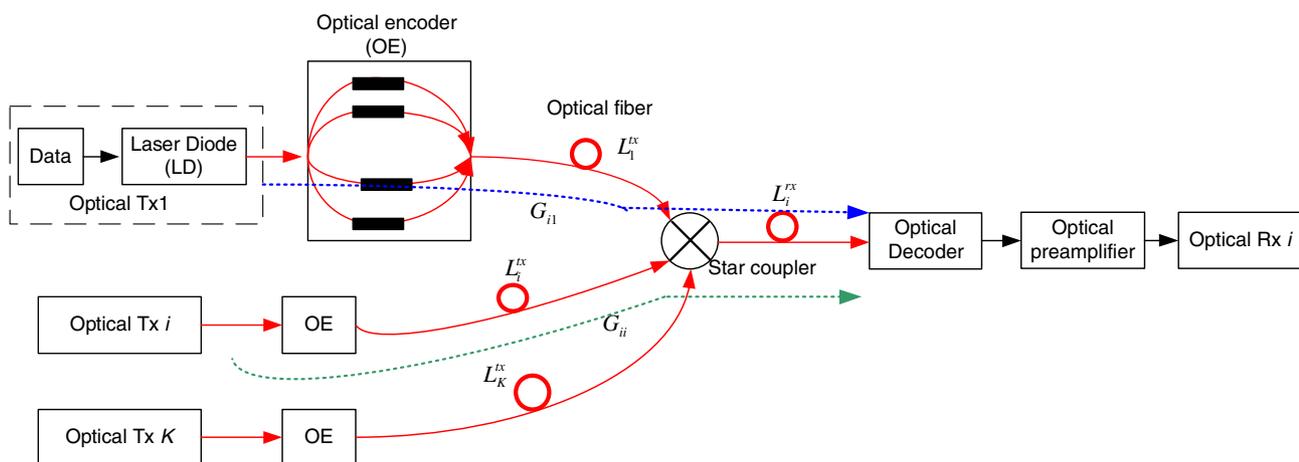


Fig. 1. An optical CDMA star network with  $K$  users.

different, as would be the case in practice, the power received from different users will be significantly different. This leads to the near far problem and power control is needed to equalize the received power from the different users accessing the network. One trivial but inefficient solution is to make the fibers connecting the different nodes to the coupler of equal length. The fiber length of each user would then be equal to that of the farthest node. However, this is certainly not the optimum arrangement in terms fiber cost or received power.

To simplify notations we assume that the transmitter and receiver pair in communication is labeled with the same subscript as shown in the figure. At the transmitter, each user's data is converted from electrical to optical (E/O) using a laser diode and then encoded optically according to prime encoding (by using a network of optical tapped delay lines) to produce the temporal OCDMA signal. The prime code consists of many blocks each containing a single pulse. For any prime number  $q$  (code weight), a code comprises of  $q$  blocks of length  $q$ . A set of code sequences of length  $N = q^2$ , derived from prime sequences of length  $q$ , was derived in [8]. Then the generated optical CDMA signal is attenuated by the fiber connecting the node to the star coupler. The star coupler broadcasts the optical signal to all nodes, so that at the receiving node we have the desired and interfering users' signals. Finally, the composite signal is decoded optically using an optical decoder that is matched to the desired user code. The network attenuation matrix  $\mathbf{G}$ , connecting all the transmitter-receiver pairs, contains the elements  $G_{ij}$  denoting the attenuation between  $j$ th Tx and  $i$ th Rx node. Then the received optical power at the input of the  $i$ th decoder due to an optical signal power of  $P_j$  from node  $j$  is then  $P_j G_{ij}$ . The matrix elements are related to the optical signal power loss due to the encoders, decoders, star coupler, connectors, and the extra intrinsic losses. Due to the severe signal power splitting loss, an optical preamplifier at the decoder output is employed. If a parallel tapped delay line is used for the encoding and decoding, then the power loss due to the optical splitters and combiners at the encoder and decoder is  $q^2$ , and the  $K \times K$  star coupler loss is  $K$  [18]. Smaller encoding and decoding power loss is possible with the ladder type implementation. In Fig. 1, the network nodes are assumed to be randomly distributed over an area centered at the star coupler and with a radius of  $L_{\min} \leq r \leq L_{\max}$ . The length of the fiber connecting the  $j$ th node to the  $i$ th node through the star coupler can be represented by,

$$L_{ij} = L_i^{\text{rx}} + L_j^{\text{tx}}, \quad \text{for } i, j = 1, 2, \dots, K, \quad (1)$$

where  $L_i^{\text{rx}}$  and  $L_j^{\text{tx}}$  are the  $i$ th and  $j$ th fiber length from the receiving and transmitting nodes to the coupler respectively.

The total received signal power is the sum of the network weighted powers from all active users. The performance of direct detection receiver can be improved significantly by using an optical preamplifier in front of the receiver. The amplified spontaneous emission (ASE)

in the optical preamplifier will be the main limiting factor (in addition to the MAI) compared to thermal and shot noise at the receiver. The spontaneous noise power at the output of the amplifier for each polarization mode is given by [19]

$$N_{\text{sp}} = n_{\text{sp}} h f_c (G_{\text{amp}} - 1) B_0, \quad (2)$$

where  $n_{\text{sp}}$  is the spontaneous emission factor typically around 2–5,  $G_{\text{amp}}$  is the amplifier gain,  $B_0$  is the optical bandwidth,  $h$  is Planck's constant, and  $f_c$  is the carrier frequency. To reduce the ASE noise power, the optical bandwidth needs to be made as small as possible. Ideally, the optical bandwidth can be set to a minimum of  $B_0 = 2R$ , where  $R$  is the bit rate.

When  $k$  users are transmitting simultaneously, the total interference at a given receiver is the superposition of  $k - 1$  different cross-correlation functions. For prime codes the average variance of the cross-correlation amplitude was found to be approximately  $\sigma^2 \approx 0.29$  independent of  $q$ , [8]. Assuming equal users' power (perfect power control) and noise free scenario, the Signal to Interference Ratio (SIR) denoted by  $\gamma$  and bit error rate (BER) are given, respectively, by [8]:

$$\gamma = \frac{q^2}{\sigma^2(k-1)} \quad (3)$$

and

$$P_E = Q\left(\frac{\sqrt{\gamma}}{2}\right), \quad (4)$$

where Gaussian approximation of the MAI distribution is assumed.

### 3. Optical power control

#### 3.1. Centralized optical power control

The target carrier to interference power ratio (CIR) required to get a certain QoS as measured by the BER for user  $i$  is denoted by  $\Gamma_i$ , which can be different from user to user and the corresponding SIR is denoted by  $\gamma_i$ . Let the transmitted optical power vector be denoted by the  $k$ -dimensional column vector  $\mathbf{P} = [P_1, P_2, \dots, P_k]^T$ . Then, optical power control can be considered as an optimization problem by finding the vector  $\mathbf{P}$  minimizing the cost function [20],

$$J(\mathbf{P}) = \mathbf{1}^T \mathbf{P} = \sum_{i=1}^k P_i. \quad (5)$$

Subject to the constraint:

$$\Gamma_i = \frac{P_i G_{ii}}{\sum_{j=1, j \neq i}^k P_j G_{ij}} \geq \Gamma_{\min} \quad (6)$$

and

$$0 \leq P_i \leq P_{\max}; \quad \forall i = 1, \dots, k, \quad (7)$$

where:

- $\mathbf{1}^T = [1, \dots, 1]$ ,
- $k$  = number of active users in the network,
- $G_{ij}$  = attenuation between transmitting node  $j$  and receiving node  $i$ ,
- $\Gamma_{\min}$  = minimum target carrier to interference power ratio,
- $P_{\max}$  = maximum transmit optical power.

Using matrix form and rearranging terms in (6) the optimum power corresponding to the minimum CIR satisfies

$$[\mathbf{I} - \Gamma_{\min} \mathbf{H}] \mathbf{P} = \mathbf{0}, \quad (8)$$

where  $\mathbf{I}$  is the identity matrix and  $\mathbf{H}$  is a nonnegative matrix with elements,

$$H_{ij} = \begin{cases} 0 & \text{when } i = j, \\ \frac{G_{ij}}{G_{ii}} & \text{when } i \neq j. \end{cases} \quad (9)$$

$\mathbf{H}$  is called the interference matrix because it quantifies the amount of interference experienced by the desired user due to the other users. From linear algebra, a solution to (8) exists only if  $1/\Gamma_{\min}$  is an eigenvalue of  $\mathbf{H}$  and its corresponding positive eigenvector  $\mathbf{P}^*$  will be the optimum power vector. According to Perron–Frobenius theorem [21], there exists a positive vector associated to the maximum eigenvalue of the nonnegative and irreducible  $k \times k$  matrix  $\mathbf{H}$ . Therefore, using the CIR constraint in (6) the solution of the power control is the eigenvector corresponding to the largest absolute eigenvalue  $|\lambda_{\max}|$  or the spectral radius  $\rho(\mathbf{H})$  of the interference matrix  $\mathbf{H}$ . Hence, the maximum achievable CIR (target CIR satisfied by all nodes at the same time) is  $\Gamma_{\max} = 1/|\lambda_{\max}|$ . When perfect power control is assumed then the received power from all users is equal at the point of reception. Consequently, taking this into account in (6) the maximum achievable CIR (MAI-limited and noise-free) in the optical star network is,

$$\Gamma_{\max} = \frac{1}{k-1}. \quad (10)$$

Therefore, from (10) and  $\Gamma_{\max} = 1/|\lambda_{\max}|$ , the maximum eigenvalue of the  $k \times k$  matrix  $\mathbf{H}$  is  $k-1$ . Furthermore, from (10) we conclude that  $\Gamma_{\max}$  is limited only by the number of nodes and the network attenuation plays no role in that respect. Assume for instance, that  $k_1$  nodes are switched off and the other  $k - k_1$  active nodes power are not changed, then the CIR of each active node will jump to  $\frac{1}{k-k_1-1}$ . Additionally, from (10) the spectral radius of  $\mathbf{H}$  is equal to the number of nodes connected to the network minus one, i.e.,  $\rho(\mathbf{H}) = k-1$ . In case of interference-limited operation (ASE noise level is very small compared to the MAI) the SIR at the optical preamplifier output and the CIR at the correlator input will be related by a constant parameter (processing gain). Therefore, the optimum power can be calculated using the maximum achievable CIR. If the optical preamplifier ASE noise level is non-negligible then the SIR constraint will be used to evaluate the

optimum power as follows. The SIR before the photodetector is given by,

$$\gamma_i = \frac{G_{\text{amp}} q^2 P_i G_{ii}}{\sigma^2 G_{\text{amp}} \sum_{j=1; j \neq i}^k P_j G_{ij} + 2N_{\text{sp}}} \geq \gamma_{\min}, \quad (11)$$

where the factor of 2 that multiplies the noise term  $N_{\text{sp}}$  accounts for the two optical polarization modes. Rearranging terms we get the transmitted optical power for the  $i$ th user,

$$P_i = \frac{\gamma_i \sigma^2}{q^2} \sum_{j=1; j \neq i}^k P_j \frac{G_{ij}}{G_{ii}} + \frac{2\gamma_i N_{\text{sp}}}{q^2 G_{\text{amp}} G_{ii}}. \quad (12)$$

Eq. (12) can be written more compactly by defining a scaled SIR and noise power as:

$$\bar{\gamma}_i = \frac{\gamma_i \sigma^2}{q^2} \quad (13)$$

and

$$u_i = \frac{2\gamma_i N_{\text{sp}}}{q^2 G_{\text{amp}} G_{ii}}. \quad (14)$$

Hence in matrix notations we have,

$$[\mathbf{I} - \Lambda \mathbf{H}] \mathbf{P} \geq \mathbf{u}, \quad (15)$$

where the matrix  $\Lambda$  is a diagonal matrix with (13) as its elements and reduces to a single element if the target SIR of all nodes is the same. Hence, the optimum transmitted power required by all users should be selected to satisfy,

$$\mathbf{P}^* = [\mathbf{I} - \Lambda_{\max} \mathbf{H}]^{-1} \mathbf{u}. \quad (16)$$

If the target SIR is increased, then higher optical power is needed which may be greater than the maximum allowable value (7). In this case, there is no solution unless we either decrease the target SIR or remove (switch off) some users from the network.

When the target SIR for all nodes is equal and the perfect power balancing is assumed then it can be shown that the maximum achievable SIR (MAI-limited noise-free) from (11) is given by,

$$\gamma_{\max} = \frac{q^2}{\sigma^2(k-1)}. \quad (17)$$

Therefore, the theoretical upper bound for SIR in optical CDMA star networks is determined by the number of nodes and the applied code correlation properties. Note that (17) is independent of the network attenuation or fiber lengths, provided all the other noise sources except MAI, even ever present quantum noise of optical systems, are neglected. However, (17) provides a basis of performance inspection in optical LANs, where system performance is indeed MAI limited. The result in (17) is equal to the case of noise-free, equal powers scenario assumed in (3), and the same behavior applies to single cell wireless communications [20]. Furthermore, the result in (17) is equal to the maximum achievable CIR in (10) multiplied by the processing gain factor of  $q^2/\sigma^2$  that is related to the applied code correlation properties. Clearly, in order to increase

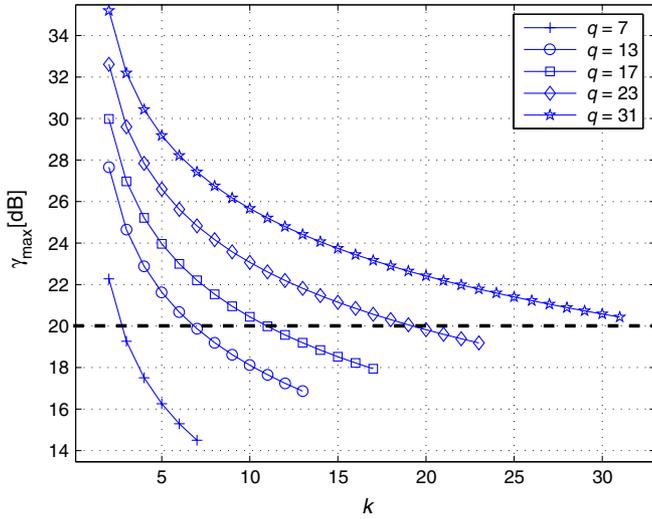


Fig. 2. Maximum achievable SIR in a star network for some prime codes.

the value of  $\gamma_{\max}$  for a fixed network size, it is required to have temporal codes with higher weights and lower cross correlation values as expected. Fig. 2 depicts the variation of  $\gamma_{\max}$  with the network size for several prime codes. The network capacity in terms of the number of supported users can be deduced from the graph, for instance, at a  $\gamma_{\max}$  value of 20 dB a network with  $q = 7$  can support only 2 users out of 7, while a network with  $q = 23$  can support 18 users out of 23.

### 3.2. Partitioned centralized optical power control

When evaluating the optimum transmission powers in (16), the network attenuation matrix  $\mathbf{G}$  connecting all the communicating node pairs is needed. This evaluation can be simplified by partitioning the star network into two parts: the access network (many to one) including the star coupler and the broadcasting network (one-to-many) after the star coupler, as shown in Fig. 3. Therefore, the network attenuation is partitioned into the loss from the Tx to the star coupler output denoted by  $g_j$  and the loss after the star coupler denoted by  $\hat{g}_i$  as labeled on Fig. 3. Hence, substituting by  $G_{ij} = g_j \hat{g}_i$  in (11), the SIR at the  $i$ th Rx node can be rewritten as,

$$\gamma_i = \frac{G_{\text{amp}} q^2 P_i}{G_{\text{amp}} \sigma^2 \sum_{j=1, j \neq i}^K P_j \frac{g_j}{g_i} + \frac{2N_{\text{sp}}}{g_i \hat{g}_i}} \geq \gamma_{\min} \quad (18)$$

and the CIR at the star coupler output from Fig. 3(a) can be written as,

$$\Gamma_i = \frac{P_i g_i}{\sum_{j=1, j \neq i}^K P_j g_j} = \frac{P_i}{\sum_{j=1, j \neq i}^K P_j \frac{g_j}{g_i}}, \quad (19)$$

where it is assumed that the noise power generated by the optical source is negligible.

**Theorem 1.** In a MAI-limited (noise-free) star coupled OCDMA network using temporal encoding with code weight

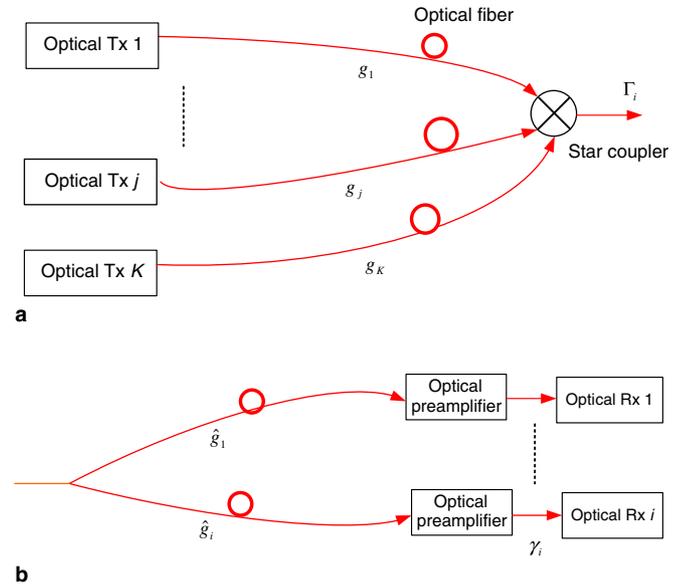


Fig. 3. Partitioning the star topology into: (a) access part; (b) broadcasting part.

$q$  and code cross-correlation variance  $\sigma^2$ , the maximum achievable SIR at the receiving nodes equals the maximum achievable CIR at the star coupler output times a processing gain factor of  $q^2/\sigma^2$ , i.e., the fiber lengths after the star coupler play no role in optimum power evaluation.

**Proof.** When the MAI is dominant then the noise term in the denominator of the SIR in (18) can be neglected (set to zero). And the result is the CIR at the star coupler output in (19) multiplied by the factor  $q^2/\sigma^2$ . This means that the maximum achievable target optical SIR at the preamplifier output can be set using the maximum achievable target CIR at star coupler output regardless of the length of the fiber lines after the star coupler. Furthermore, by using  $G_{ij} = g_j \hat{g}_i$  in (9) the interference matrix elements are given by,

$$H_{ij} = \begin{cases} 0 & \text{when } i = j, \\ \frac{g_j}{g_i} & \text{when } i \neq j. \end{cases} \quad (20)$$

Therefore, it is clear that the fiber cables after the star coupler do not play a role in the MAI encountered by the receiving nodes.  $\square$

A consequence of the above theorem is that users need not to know the other ongoing communicating pairs, i.e., from which  $j$ th source node to which  $i$ th destination node. In other words, the total network matrix  $\mathbf{G}$ , which depends on the current ongoing connections, is not required but only the access part of it. A simple algorithm for power control is then to equip each user with a lookup table containing the fiber lengths of all other users and the status of each user either ON or OFF (but not to which node it is currently sending data) to evaluate its optimum power. The lookup table content is updated regularly. Each time

a user status is changed the optimum power level is recalculated if the target CIR is different from the maximum achievable CIR.

*Special case:* Let the fiber lengths from the transmitting nodes to the star coupler be equal, i.e.,  $g_1 = g_2 = \dots = g_K = g$ , then the interference matrix in (20) is given by,

$$H_{ij} = \begin{cases} 0 & \text{when } i = j, \\ 1 & \text{when } i \neq j. \end{cases} \quad (21)$$

Then, all users will encounter the same interference level regardless of the differences in the fiber lengths after the star coupler. This means that the users will transmit at an equal power level.

### 3.3. Distributed optical power control

The centralized power control suffers from the main drawback of the need of a central node with all information about link gains (fiber attenuation, star coupler attenuation, etc.) and which transmitter to which receiver ongoing transmission. For distributed power control, only local information (previous power and estimated CIR or SIR) is needed to set the optimal optical power at the specific transmitting node. The optical power for the  $i$ th node is updated according to,

$$P_i(n+1) = \Psi(P_i(n), \Gamma_i(n)), \quad (22)$$

where  $\Psi(\cdot)$  is called the interference function. Table 1 summarizes several distributed power control algorithms. In the table the algorithms are initialized with a power vector such that  $\mathbf{P} = \mathbf{P}_0$ ;  $\mathbf{P}_0 > \mathbf{0}$ . A brief study of these algorithms for wireless communications and their convergence properties can be found in [20]. The power updating algorithms in the table requires the knowledge of the CIR at the intended node. We will consider only the DBA algorithm (first entry in Table 1) although the same can be applied to all other distributed algorithms. Let us inspect two techniques for CIR estimation. In the first method, each receiving node measures the total received optical power  $P_{rx}(n)$  (signal plus interference) before the decoders and reports the value back to the sending node as a side information. The sending node  $i$  then uses the following formula to estimate the CIR,

$$\tilde{\Gamma}_i(n) = \frac{P_i(n)G_{ii}}{P_{rx}(n) - P_i(n)G_{ii}}. \quad (23)$$

The disadvantage of this technique is that the power control algorithms are no longer distributed because  $G_{ii}$  is needed for the CIR estimation.

The other interesting technique is based on network partitioning presented in the previous section. From Theorem 1, the maximum achievable SIR at the receiving nodes is equal to the maximum achievable CIR at the output of the star coupler times  $q^2/\sigma^2$ . Therefore, the technique is based on measuring and sending back the total power at the output of the access part of the network  $P_{star}(n)$  to all nodes. Then, the transmitting nodes estimate the CIR at the intended receiver using (19), rewritten here as,

$$\tilde{\Gamma}_i(n) = \frac{P_i(n)g_i}{P_{star}(n) - P_i(n)g_i}, \quad (24)$$

where  $P_{star}(n) = \sum_{j=1}^K P_j(n)g_j$ . By testing this estimate on numerical values with randomly initialized positive power vector in the DBA algorithm we found that the algorithm converges to a vector that satisfies the maximum achievable CIR for all users. This can be described by substituting this CIR estimate in the DBA algorithm and after simple algebra we get,

$$P_i(n+1) = \beta \left[ P_i(n) + \sum_{j=1, j \neq i}^K \frac{g_j}{g_i} P_j(n) \right]. \quad (25)$$

From (25), the CIR estimate in (24) is equivalent to adding a weighted sum of the other users' power. The weighting is proportional to the ratio between the interfering to the desired user optical power attenuation. In matrix form, (25) is written as,

$$\mathbf{P}_{n+1} = \beta[\mathbf{I} + \mathbf{H}]\mathbf{P}_n = \beta^{n+1}[\mathbf{I} + \mathbf{H}]^{n+1}\mathbf{P}_0. \quad (26)$$

It is important to note that (26) do not converge to a fixed point but the resulting power vector satisfies the maximum achievable CIR given in (10) as will be shown below. The power updating procedures are assumed to occur synchronously, i.e., at the same time instants at all nodes.

The condition for guaranteed convergence of the DBA algorithm can be evaluated as follows. Because  $[\mathbf{I} + \mathbf{H}]^{n+1} = \mathbf{V}\Sigma\mathbf{V}^T$  where  $\mathbf{V} = [\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_K]$  the  $\mathbf{v}_i$  is the  $i$ th eigenvector of matrix  $\mathbf{H}$  and  $\Sigma$  is a diagonal eigenvalues matrix, then,

$$\begin{aligned} \mathbf{P}_{n+1} &= \beta^{n+1}\mathbf{V}\Sigma^{n+1}\mathbf{V}^T\mathbf{P}_0 \\ &= \beta^{n+1}[1 + \rho(\mathbf{H})]^{n+1}\bar{\Sigma}^{n+1}\mathbf{V}^T\mathbf{P}_0, \end{aligned} \quad (27)$$

where  $\rho(\mathbf{H})$  is the spectral radius of  $\mathbf{H}$  and  $\bar{\Sigma}$  is a diagonal matrix with all its elements less than or equal to one. For large  $n$ , Eq. (27) can be approximated as,

$$\mathbf{P}_{n+1} \simeq \beta^{n+1}[1 + \rho(\mathbf{H})]^{n+1}\mathbf{v}_1\mathbf{v}_1^T\mathbf{P}_0, \quad (28)$$

where  $\mathbf{v}_1$  is the eigenvector associated with largest eigenvalue  $\rho(\mathbf{H})$ . Since  $\mathbf{v}_1^T\mathbf{P}_0$  is a constant scalar and assumed

Table 1  
Distributed power control algorithms

Name	Algorithm
Distributed balancing algorithm (DBA)	$P_i(n+1) = \beta P_i(n)(1 + \frac{1}{\tilde{\Gamma}_i(n)}); \beta > 0$
Distributed power control (DPC)	$P_i(n+1) = \beta(n) \frac{P_i(n)}{\tilde{\Gamma}_i(n)}; \beta(n) = \frac{1}{\max\{P_i(n)\}_{i=1}^K}$
Distributed constrained power control (DCPC)	$\min\{P_{max}, \Gamma^T \frac{P_i(n)}{\tilde{\Gamma}_i(n)}\}$
Fully distributed power control (FDPC)	$P_i(n+1) = \frac{\min(\tilde{\Gamma}_i(n), \bar{\xi})}{\tilde{\Gamma}_i(n)} P_i(n); 0 < \bar{\xi} < \infty$

here equal to one without loss of generality, therefore, the solution of the DBA algorithm is the eigenvector associated with the maximum eigenvalue of  $[\mathbf{I} + \mathbf{H}]$  multiplied by a scaling factor. Then convergence of the DBA algorithm is conditioned on,

$$[\beta(1 + \rho(\mathbf{H}))]^{n+1} < \infty. \tag{29}$$

If  $\beta < \frac{1}{K}$  then the resulting power vector is decreasing toward zero otherwise it is increasing toward infinity. To overcome this problem, the DBA is modified so that after each updating step in (26) the updated vector is normalized by its maximum value, i.e.,

$$\mathbf{P}_{n+1} = \frac{\mathbf{P}_{n+1}}{\|\mathbf{P}_{n+1}\|_\infty}, \tag{30}$$

where  $\|\mathbf{P}_{n+1}\|_\infty$  is the infinity norm. In order to prove that the resultant power vector satisfies the maximum achievable target CIR, we use  $\mathbf{H}\mathbf{v}_1 = \rho(\mathbf{H})\mathbf{v}_1$  and (28) in (19) we get,

$$\Gamma_i = \frac{\alpha_{n+1} \mathbf{1}_i^T \mathbf{v}_1}{\alpha_{n+1} \mathbf{1}_i^T [\mathbf{H}\mathbf{v}_1]} = \frac{1}{\rho(\mathbf{H})}, \tag{31}$$

where  $\mathbf{1}_i$  is a vector of zeros with the  $i$ th element equal to one and  $\alpha_{n+1} = \beta^{n+1} [1 + \rho(\mathbf{H})]^{n+1}$ . Therefore, we can apply arbitrary scaling factors to the resultant power vector without violating the target CIR requirements which helps in setting the power of the optical sources to practical levels.

#### 4. Numerical results

First we consider a two user scenario each with a code of length of 49 ( $q = 7$ ). The main purpose of this setup is to investigate the system effects on an optical Gaussian pulse

data as it propagates through the fiber network taking into account several fiber impairments. As shown in Fig. 4, the network is simulated using VPItransmissionMaker tool that models the fiber by solving the nonlinear Schrödinger (NLS) equation describing the propagation of linearly polarized optical waves in fibers using the split-step Fourier method. The following typical parameters are used in simulations: Data rate = 500 Mbps, fiber dispersion coefficient =  $16 \times 10^{-6}$  [s/m<sup>2</sup>], slope of the dispersion coefficient =  $0.08 \times 10^3$  [s/m<sup>3</sup>], fiber nonlinear refractive index =  $2.6 \times 10^{-20}$  [m<sup>2</sup>/W], effective core area of the fiber =  $80.0 \times 10^{-12}$  [m<sup>2</sup>]. The fiber lengths are indicated on the diagram. A Gaussian shaped optical pulses with the full width at half maximum power (FWHM) of 10.2 ps are generated using a pulsed laser source which is directly modulated from the data source of Pseudo Random Binary Sequence generator (PRBS). A random optical delay at the second-user encoder output is inserted to account for the asynchronous operation. To overcome the severe optical signal degradation due to splitting losses an optical amplifier with a gain of 10 dB is employed. The optical filters are first order, Gaussian shaped with bandwidth ~25 GHz centered at 193.1 THz and optimized recursively for minimum BER. An optical receiver that includes an optical photodetector with responsivity of one, and a third-order low pass Bessel filter with the cutoff frequency of 0.7 times the data rate is applied for both users. Thermal and shot noise of the receiver are set to zero. Furthermore, synchronization and clock recovery is performed to set the sampling time instants for the decision circuit. The eye pattern for both users is plotted in Fig. 5 when no power control is used, and both users transmit at 0 dBm. We note that the eye opening is clear for the first-user because it is closer to the central node while the second-user

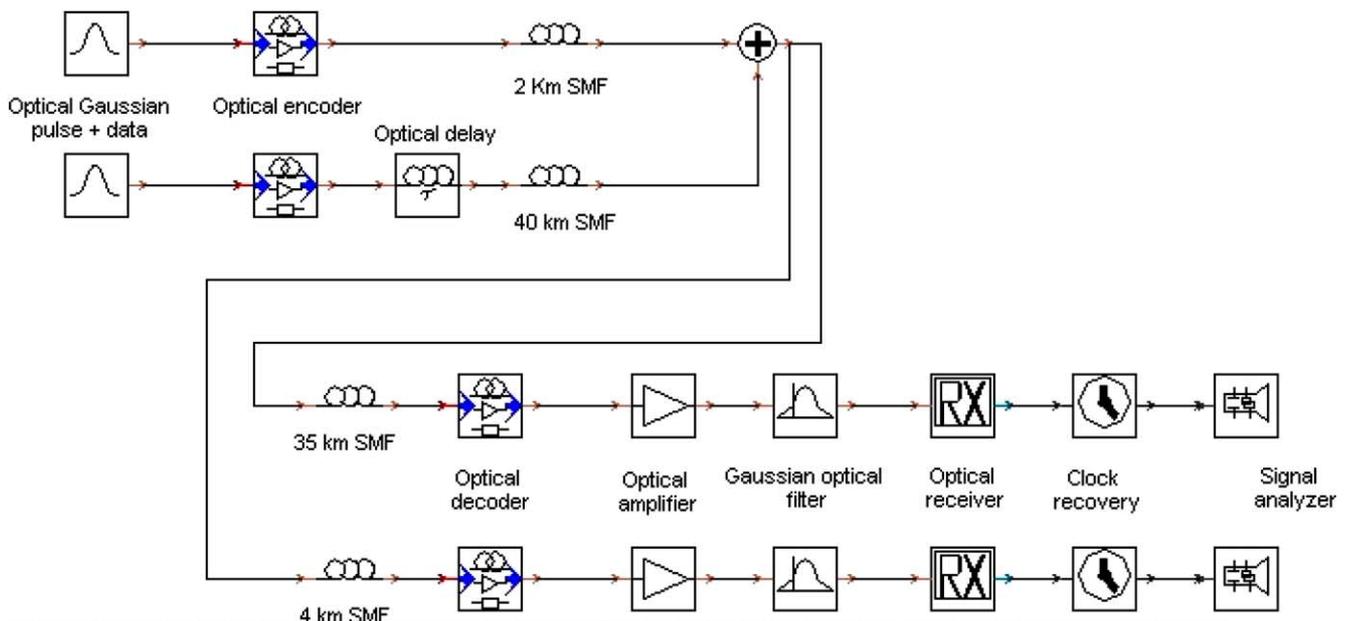


Fig. 4. Simulation setup for two user.

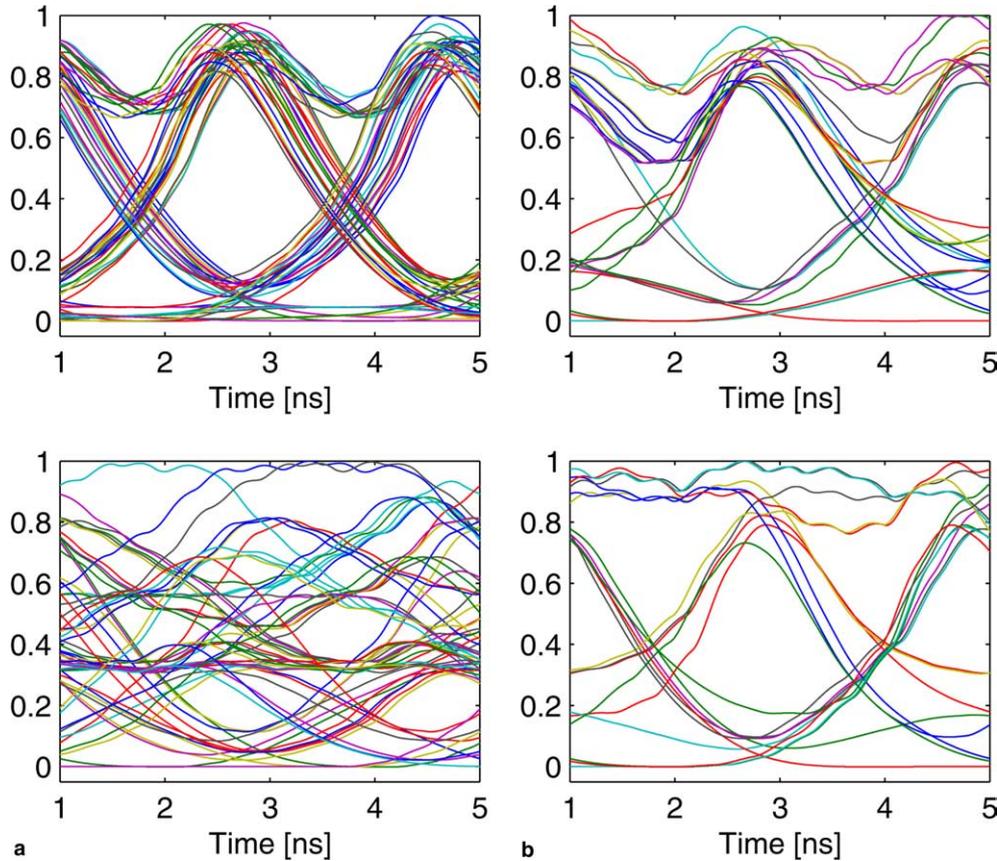


Fig. 5. Eye diagram for both users: (a) without power control; (b) with power control.

eye pattern is closed indicating a severe performance degradation. Evaluating the optimum power vector using (16) results in  $[-7.3, 0]$  dBm. Then the eye pattern shown in Fig. 5(b) shows that both users have similar characteristics which reflects the comparable performance for both users. It should be noted that although the fiber dispersion and nonlinearity are included here while they are neglected in the theoretical analysis, the results will not be affected due to the applied low data rates, short fiber lengths and low optical powers. Furthermore, the optical power control is concerned with the optimum leveling of the applied power of the different users rather than the exact signal shape which is affected by the dispersion and nonlinearity impairments. To include such effects they can be added as power penalties to the network attenuation matrix before calculating the optimum power levels.

Next, we consider a network with 31 nodes ( $q = 31$ ) that are uniformly distributed over an area with a radius of  $2 \leq r \leq 50$  [km] and the star coupler is at the area center. A star coupler excess loss of 0.2 dB, and the fiber attenuation coefficient is 0.2 dB/km at the operating wavelength of 1550 nm. Bit rate of 2.5 Gbps and a maximum allowable transmit laser power of 20 dBm for all nodes is assumed. We neglect the effect of dispersion by assuming the use of dispersion shifted fiber links. Fiber nonlinearity is also neglected because of the relatively short fibers, low data rates and low optimum power levels encountered. The maximum

achievable SIR from (17) if all other users are active is 20.43 dB (BER of  $7.4 \times 10^{-8}$ ). By comparing the Tx-to-coupler fiber lengths, coupler-to-Rx fiber lengths and the corresponding optimum power calculated using (16), as expected the nodes with long Tx-to-coupler fibers (such as node 1, 5 and 13 in Fig. 6(a)) will transmit the highest power. On the other hand, nodes with short Tx-to-coupler fibers will transmit the smallest power. The mean laser power for all users is approximately 12 dBm when power control is applied as compared to 20 dBm without power control. Fig. 7 shows the BER for the longest and the shortest Tx-to-coupler nodes. Without power control, the performance of the closer nodes to the coupler is many orders of magnitude better than that of the far nodes, by using power control these nodes will get the same quality of service if the target SIR is set to the same value. Since there is a possibility that only one user is far from the star coupler and all other users are at shorter and comparable distances so they have the same high performance. Therefore, we plot the mean bit error rate for all active users in the network as shown in Fig. 8. The network capacity or number of supported nodes satisfying the required SIR is needed along with these curves for better judgment of the quality of service. The network capacity is shown in Fig. 9, which is the same as in Fig. 2 when power control is applied. For the target SIR of 15 dB, all users can be supported with or without power control. As the target SIR is

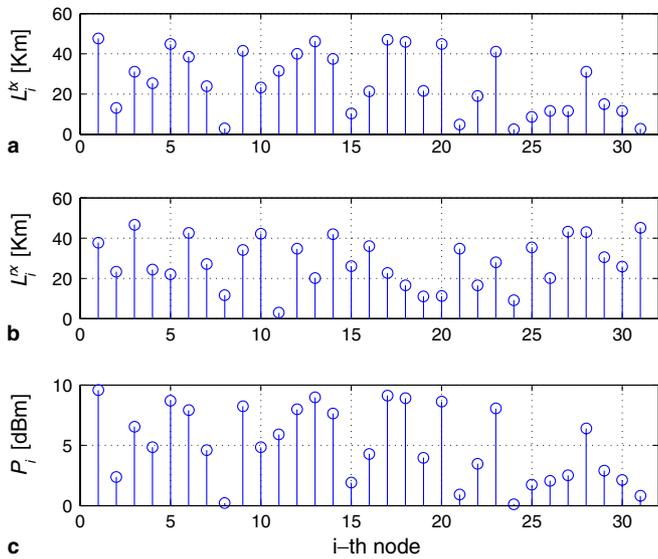


Fig. 6. (a) Fiber lengths from transmit nodes to coupler; (b) fiber lengths from receive nodes transmit node to coupler; (c) optimum laser powers.

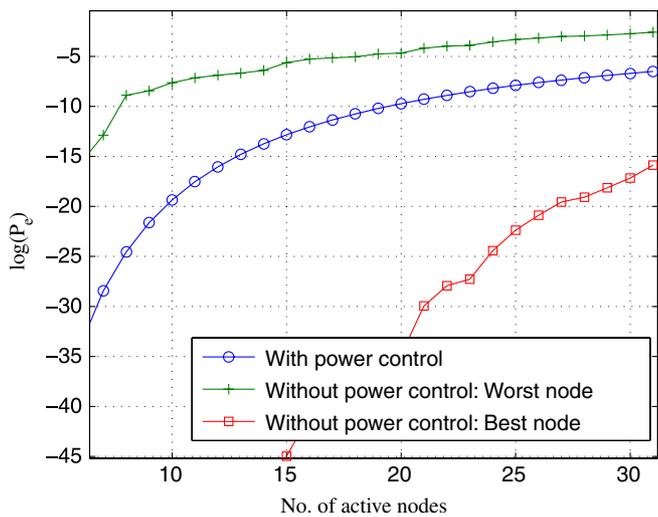


Fig. 7. Bit error rate for a Prime coded network with and without power control for the worst node and the best node. Prime number of 31.

increased to  $\sim 20$  dB (BER of  $2.8 \times 10^{-7}$ ) the power controlled regime still supports all users, while only 11 users can be supported if no power control is applied. If the target SIR is greater than 20 dB, the number of supported users decreases gradually for both cases with approximately two times number of users supported in the power controlled scenario.

### 5. Conclusions

In this paper, we discussed the application of power control algorithms to optical CDMA. We considered a star coupled network with uniformly distributed user nodes around the star coupler. Fiber attenuation, star coupler splitting loss, encoders–decoders splitting loss and the

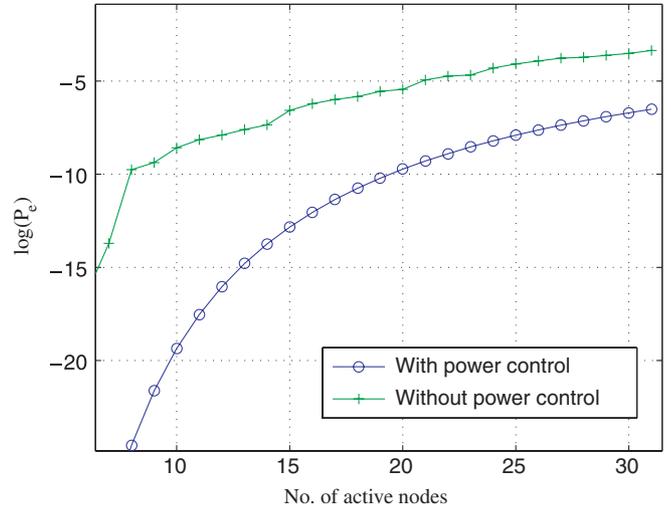


Fig. 8. Mean bit error rate for a Prime coded network with and without power control. Prime number of 31.

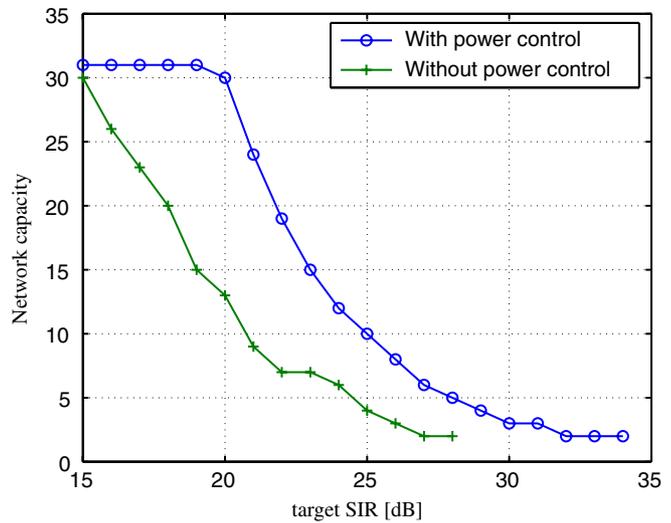


Fig. 9. Number of supported nodes satisfying the target SIR in a Prime coded network with and without power control. Prime number of 31.

intrinsic losses are taken into account. Also, an optical pre-amplifier is employed at the receiver input to decrease the effect of network attenuation on the transmitted optical signal. Amplified spontaneous emission noise is considered as the main noise source along with the multiple access interference. We found that the network performance is upper bounded by a function that is related to the number of nodes and the correlation properties of the employed code. Furthermore, we showed that by using optimum optical transmit powers, the capacity of the network in terms of the number of supported users could be enhanced significantly. Then, we showed that the fibers after the star coupler are irrelevant to the optimum power evaluation. We have also demonstrated that by partitioning the network to access and broadcast parts the SIR at the receiving nodes and the CIR at the star coupler output are related

by a gain factor that is in turn related to the code correlation properties. Moreover, we used the distributed power control algorithms to set the optimum power without the need for prior knowledge of transmit power information about other users accessing the network.

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